Algebraic reasoning

Cats and Canaries

Have you ever stared at a math problem and decided that there is no way you are ever, ever going to solve it. When you first work on the problems below, they could make you feel that way. But if you try a guess and check approach or draw a picture, you will find that they can be surprisingly easy. The title of this activity comes from the animals in the first problem. (You may want to try the *Guess and Check* and *Flowerpots* activities before this one.)

These problems can be written as a *system of two simultaneous equations*. There are many real-world applications for this type of system. For example, suppose you want to combine chocolate peanuts that cost 60¢ a pound with cashew nuts that cost $1.10 a pound, to make a mixture that cost $1.00 a pound. How many ounces of each do you put into each pound of mixture? What if the most you want to pay is 75¢ a pound?

MATERIALS

- pencil and paper
- calculators
- activity cards

WHAT'S THE MATH?

Algebraic language; simultaneous equations; proportional reasoning.

How

• Have your group of 3 to 6 people select a puzzle. Pass out the clue cards. If there are extras, it's fine for some people to have two cards.

• You may read the information on your card aloud as many times as you wish. But you cannot give your card to someone else to read.

• Work together in your group to figure out the problem. Use any method or tools that you think will help, such as drawing diagrams, using blocks and beans, making guesses, or a combination of ways.

• When you have found a solution, check to see that it fits all of the clues.

• Compare your methods with other groups. Do you have a favorite?

• Try some other problems.

Extension

- Make up some problems for another group to try. Make sure you try them first to see that they work.

Cats and Canaries

Activity **cards**

Cats & Canaries

- Ms. Lang keeps cats and canaries.

  She has 25 heads to pet.

  How many cats does she have?

Cats & Canaries

- Ms. Lang keeps cats and canaries.

  She counted three times as many cat paws as canary feet.

  How many cats does she have?

Cats & Canaries

- Ms. Lang keeps cats and canaries.

  The number of cat paws and the number of canary feet share common factors.

  How many cats does she have?

(*Note:* A *factor* is a number which divides into another number evenly.)

Cats & Canaries

- Ms. Lang keeps cats and canaries.

  The total number of cat paws is a multiple of 5.

  How many cats does she have?

Cats & Canaries

- Ms. Lang keeps cats and canaries.

  The total of the number of cat paws and canary feet is divisible by 2, 4, 8, 10, 20, 40, and 80.

  How many cats does she have?

Cats & Canaries

- Ms. Lang keeps cats and canaries.

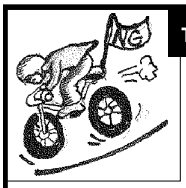
  The total number of canary feet is a multiple of 5.

  How many cats does she have?

Activity

cards

The Ng Family

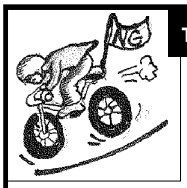


The Ng Family

- There are 6 children in the Ng family.

  Each child has either a bicycle or a tricycle.

  How many bicycles does the family have?

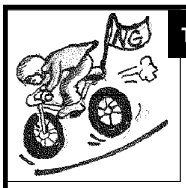


The Ng Family

- There are 6 children in the Ng family.

  No child has both a bicycle and a tricycle.

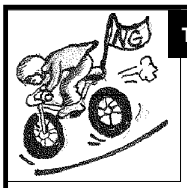
  How many bicycles does the family have?



The Ng Family

- The oldest child says there are exactly 14 wheels altogether.

  How many bicycles does the family have?

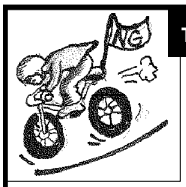


The Ng Family

- Next year Dante will be old enough to ride a bicycle.

  Then there will be a total of 13 wheels.

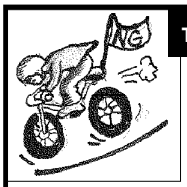
  How many bicycles does the family have now?



The Ng Family

- Only the children have bicycles or tricycles in the Ng family.

  How many bicycles does the Ng family have?



The Ng Family

- The number of tricycles in the Ng family is a factor of the number of bicycles.

  How many bicycles does the family have?

Starfish and Octopi

Activity **cards**

Starfish & Octopi

- Mr. Ruiz keeps starfish (with five arms each) and octopi (with 8 arms each).

  How many starfish does Mr. Ruiz have?

Starfish & Octopi

- Mr. Ruiz has 24 mouths to feed.

  How many starfish does he have?

Starfish & Octopi

- Mr. Ruiz's son counted a total of 150 arms on the sea creatures.

  How many starfish does Mr. Ruiz have?

Starfish & Octopi

- The difference between the number of octopus arms and the number of starfish arms is 10.

  How many starfish does Mr. Ruiz have?

Starfish & Octopi

- The number of starfish arms is a multiple of the number of octopus mouths.

  How many starfish does Mr. Ruiz have?

Starfish & Octopi

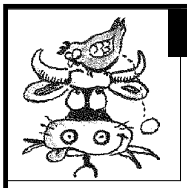
- The difference between the number of starfish and octopi is less than 10.

  How many starfish does Mr. Ruiz have?

Activity

cards

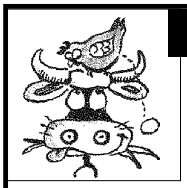
Farmer MacDonald



Farmer MacDonald

- Farmer MacDonald keeps cows and chickens.

  How many cows does she have?



Farmer MacDonald

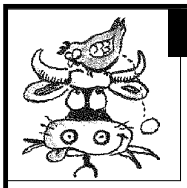
- Altogether, Farmer MacDonald has 45 cows and chickens to feed.

  How many cows does she have?

Farmer MacDonald

- The vet told Farmer MacDonald that these creatures have a total of 110 feet.

  How many cows does she have?



Farmer MacDonald

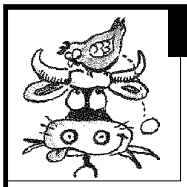
- There are four times as many cow feet as there are cows.

  How many cows does Farmer MacDonald have?

Farmer MacDonald

- There is a difference of 30 between the number of chicken feet and the number of cow feet.

  How many cows does Farmer MacDonald have?



Farmer MacDonald

- The number of cows on Farmer MacDonald's farm is divisible by 2.

  How many cows does she have?

Cycles

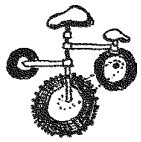
Activity **cards**

Cycles

- Mary Ellen wants to make unicycles, bicycles, and tricycles with spare wheels and bike seats.

  How many of each can she make?

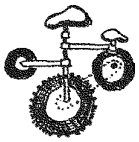
Cycles



- Mary Ellen has 26 seats to use to make unicycles, bicycles, and tricycles.

  How many of each can she make?

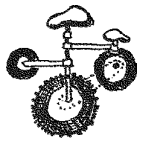
Cycles



- Mary Ellen has 60 wheels to use to make unicycles, bicycles, and tricycles.

  How many of each can she make?

Cycles



- Mary Ellen can make more bicycles than other types of cycles.

  How many of each type can she make?

Cycles

- The number of wheels Mary Ellen puts on bicycles is close to the number of wheels she puts on tricycles.

  How many of each type of cycle can she make?

Cycles

- Mary Ellen does not use very many wheels to make unicycles.

  How many of each type of cycle does she make?

Cats and Canaries

Algebra Notes

These problems can be written as two *simultaneous equations* with two unknowns. The goal is to find a solution that is true for both equations.

For example, in *Cats and Canaries*, if we let x stand for the number of cats and y for the number of canaries, we know the following. The total number of animals is 25, that is

$$x + y = 25.$$

We also know that the total number of legs is 80. Each cat has four legs and each canary has 2, so the number of cat legs is 4 times the number of cats, or 4x, and the number of canary legs is 2 times the number of canaries, or 2y, so

$$4x + 2y = 80.$$

Now we have two equations: $x + y = 25$ and $4x + 2y = 80$.

There are various approaches to solving these two equations for x and y. One idea is to find what x is in terms of y using just one of the equations, and then to substitute that result into the other equation.

Here we can look at

$$x + y = 25.$$

Subtracting y from both sides, we have

$$x + y - y = 25 - y;$$

thus

$$x = 25 - y.$$

Substituting this result for x in the other equation, that is, replacing x with 25 – y, we have

$$4(25 - y) + 2y = 80$$

$$4 \cdot 25 - 4y + 2y = 80;$$

thus

$$100 - 2y = 80.$$

Adding 2y to both sides, we have

$$100 - 2y + 2y = 80 + 2y;$$

thus

$$100 = 80 + 2y.$$

Subtracting 80 from both sides, we have

$$100 - 80 = 80 - 80 + 2y;$$

thus

$$20 = 2y.$$

Dividing by 2, we have

$$\frac{20}{2} = \frac{2y}{2};$$

thus $10 = y$, which is the same as $y = 10$.

Now we know the number of canaries is 10. Subtracting 10 from 25, that means there are 15 cats. To check, we need to see if

$4 \cdot 15$, the number of cat legs plus

$2 \cdot 10$, the number of canary legs is equal to 80.

Well,

$4 \cdot 15 = 60$ and $2 \cdot 10 = 20$
and their total is 80.

The answers check. ■